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Cylindrically Symmetric Petrov Type I Magnetostatic Cosmological Framework

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ABSTRACT :

This paper discusses a magnetohydrostatic cosmological model by making use of Marders Cylindrically Symmetric Metric. The model thus contains a perfect fluid matter content, together with an impinging magnetic field, where the vacuum part is of Petrov type-I degenerate. This analysis proves the model to be heterogeneous. We will delve deeply into the spatial and geometrical characteristics of the model, allowing us to comprehend its structure and dynamics. Moreover, we elaborate on the Newtonian analogue of the force in this cosmology and the objectives it can achieve. This work aids our understanding of cosmological models with a magnetic field.

Key Words: Inhomogeneous, magnetohydrostatic, curvature tensor, pressure, symmetric, density.

1. INTRODUCTION

The introduction of magnetic fields in cosmological models has long been a important subject of research with the first work, by quarter of century ago, being done by Khalatnikov in 1967 [12]. • Anisotropic magnetohydrodynamic model [24] Roy and Prakash explained the nature of magnetic fields in the universe and their dynamics. Static magnetic universe (Patel and Vaidya) [17], Inhomogeneous cosmological models for dust (Szekeres) [25] Fluid dynamics is a type of galactic observations [35, 36] with strong magnetic fields, which causes the viscous terms. Magnetic fields play a key role in evolving cosmic structures. In this work, Cowling and Wróbel [6, 6a, 32] focused on magnetic field behaviour in stars and showed some of these magnetohydrodynamic properties. Newman examined cosmological models with magnetic fields (Zeldovich [34], Thorne [28]). Both Shikin [26] and Monaghan [15] investigated uniform axially symmetric Einstein-Maxwell model with ideal fluid and magnetic field respectively. A Bianchi type-I model consisting of a homogeneous magnetic field was first introduced by Jacobs [11, 11a] but was shortly improved by De [7] with the help of different technique. This work was furthered by Tupper when he added in Einstein-Maxwell fields with non-vanishing electric fields [30, 31]. Magnetostatic models with dust and disordered radiation have been investigated by Bali and Jain [1]. In terms of the models studied in this paper, in a separate field, Singh and Yadav [27] and Yadav and Purushottam [33] considered spatially homogeneous models with a perfect fluid and an electromagnetic field, with the help of Marder's cylindrically symmetric metric[14]. Other works regarding tohoto field include 9 [22] [3] [4] [21]. For [20,21], Tolman [29], Paul [18,19], Rao et al. [23].

We develop a magnetohydrostatic cosmological model in this study, using Marder's cylindrically symmetric metric [14]. This is a model of an anisotropic material in presence of a perfect fluid and an incident magnetic field under the influence of a Petrov type-I degenerate free gravitational field. The resulting model is inhomogeneous. We discuss its physical and geometrical features, investigate the Newtonian counterpart of force in the system and give some of the cosmological implications.

2. The Field Equations

We take the cylindrical symmetric metric of Marder [14] as:

(1) [d{s}^{2}=A^{2}(d{x}^{2}-d{t}^{2})+B^{2}{d}y^{2}-C^{2} d{z}^{2}]

where (A), (B), and (C) are functions of (x) only. Equations (2.2) - (2.5) provide the non-zero components of the Ricci tensor; and the scalar curvature is given by (2.6). This gives the following non-vanishing components of the Weyl conformal curvature tensor (C_{hijk}) [2.7 2.8 2.9 2.10].

The distribution consists of a perfect fluid with infinite electrical conductivity and a magnetic field that is electrically neutral. The energy-momentum tensor composites linearly, and we see in (2.14) the relation Tμν=∑ i Tμvi. This tensor defined in equation, 2.15 is found to be, (\mu) is magnetic permeability, (\rho) is fluid density, (p) is pressure formtypescript code This satisfies the flow vector (u^i) according to ( )}

[g_{ij} u^i u^j = -1]

Here we are in comoving coordinates, (u^1 = u^2 = u^3 = 0), where we get (u^4 = -A) and (u_4 = 1/A). Magnetic force: here the magnetic field is along the (x)-axis, thus, (F_{ij}) is only (F_{23}) non-zero component of electromagnetic field tensor. From Maxwell's first equation, it follows that (F_{23}) is constant, which we denote (H).

The electromagnetic tensor as defined by Lichnerowicz

3. Solution of the Field Equations

Equations (2.22) − (2.25) provide us with a system of four equations for five unknowns: the metric potentials (A), (B), (C), the density (\rho), and the pressure (p); such a system is underdetermined. The solution turns out to be over-determined, hence, proceeding we add an extra condition as we assume spacetime to be Petrov type-I degenerate in (y)- and (z)-directions. This condition comes from the (2.13) components of the Weyl tensor, yielding:

(3.1[C_{1212} = C_{1313})

Equation (3.1) is identically satisfied, and we have distinct metric potentials (A),(B), and (C). If we subtract equation (2.23) from (2.22), we have:

(3.2)[frac{A"}{A} frac{B"}{B} = 0]

Moreover, from equations (2.23) and (2.24), we have the following:

And then,[(3.3) B′B+C′C−A′B′AB−A′C′AC=0]

By plugging equations (3.1) till (3.3), we get:

(3.4)A′′A=0[\frac{A"}{A} = 0 \tag{3.4}]

Since(B≠C), then we have from (3.4):

[A = L \tag{3.5}]

where (L) is a constant. Substituting (3.5) into (3.2) gives:

[B" = 0 \tag{3.6}]

Integrating equation (3.3), we obtain:

[B' C - B C' = D \tag{3.7}]

where D is constant of integration Let us define (\lambda = B C) and (\nu = B / C). From (3.7), we get:

[\lambda' = D \tag{3.8}]

Based on (3.7) and (3.8), we have the derivation of:

Equation (3.9) : [\nu' = \beta \nu]

Integrating (3.9) yields:

Equation 3.10 \nu = αeβx

where (\alpha) and (\beta) are the integration constants. Expanding (3.9) and (3.10), we arrive at:

و عنه الكذاN↔(3.11)λ=(Dxd+N)α−−√eβxd/2

Integrating (3.11) gives:

Linkable Functions: [B = \sqrt{\alpha} e^{\beta x / 2} (D x + N)^{1/2}, \quad C = \frac{(D x + N)^{1/2}}{\sqrt{\alpha} e^{\beta x / 2}} \tag{3.12, 3.13}]

(N) is a integration constant, (N) The metric (2.1) becomes:

(3.15) [ds^2 = L^2(dx^2 − dt^2) + αe^βx(Dx + N)dy^2 − (Dx + N)αe^βx dz^2 Applying Ghost Spring Action in Warp-Spring]

The metric assumes the form after applying the coordinate transformations (X=x), (Y=y), (Z=z), (T=L t):

(3.20) [ds^2 = L^2 (dX^2 - dT^2) + \alpha e^{\beta X} (D X + N) dY^2 − (D X + N) α e^{\beta X } dZ^2.]

4. Some Physical and Geometrical Properties

Cylindrically symmetric Petrov Type I magnetostatic cosmological model, is a spacetime with specific mathematical properties based on general relativity Cylindrical symmetry means that the answer does not depend on the angle φ and on the distance from the axis of symmetry ρ, which we can express in (ρ, φ, z, t) coordinates. The metric g_μν is diagonal, depends on ρ, and satisfies the field equations of Einstein. A Weyl tensor is referred to as Petrov Type I if its principal null directions are, in the most general case, four-fold degenerate and distinguished by four separate, (non-degenerate) eigenvalues (which makes this type distinct from simpler type D or type O).

The word "magnetostatic" stands for a static magnetic field, implemented through the electromagnetic field tensor F_μν with the non-trivial components oriented along the symmetry axis. The energy momentum tensor T μ ν consists of terms for the magnetic field and for a perfect fluid (or anisotropic matter) and is given by T μ ν = (ρ + p) uμ uν + p g μ ν + F μα F ν α − (1/4) g μ ν F α β F α β, where ρ is the energy density, p is the pressure, and u μ is the four velocity. While we can do whatever we like with the metric, this relationship — Einstein's equations, R_μν - (1/2)Rg_μν = 8πT_μν — connects the metric to things we can measure.

Two important classes of mathematical objects include those that capture aspects of local curvature (the Ricci tensor R_μν and scalar R), and those that determine the local tidal interaction between bodies (the Weyl tensor C_μνρσ). From the field equations, one obtains differential equations for the metric components, that are solved using appropriate boundary conditions. The strength of the magnetic field — obtained from Maxwell's equations $\nabla_\mu F^{\mu\nu} = 0$ — affects $T_{\mu\nu}$ and thus the geometry of spacetime. The motion of a free particle is described by the geodesic equations $d^2x^\mu/d\tau^2 + \Gamma^\mu_{\nu\lambda}(dx^\nu/d\tau)(dx^\lambda/d\tau) = 0$, which include geodesic equations, gravitational fields, and magnetic fields.

This might yield Bessel or exponential functions for the metric components (indicating radial dependence). Depending on what parameters you choose, you will either have singularities or horizons. As a concrete model, this anisotropic box is a simple mathematical tool to assess the gravitational and electromagnetic coupling of magnetized cosmological spacetimes, thus applicable in the context of theoretical cosmology.

5. Newtonian Analogus of Force

A Newtonian Analogue of Force for a Cylindrically Symmetric Petrov Type I Magnetostatic Cosmological Model

This is a cylindrically symmetric Petrov Type I magnetostatic cosmological model spacetime, that is invariant under rotations and translations along a rotationally symmetric central axial direction, with coordinates (ρ, ϕ, z, t). The Petrov Type I (at least with respect to a Weyl tensor) is that it is one with four distinct principal null directions. The magnetostatic limit means we are under the influence of a time-independent magnetic field along the z direction, which contributes to spacetime curvature.

The metric $ds^2 = -e^{2\Phi(\rho)}dt^2 + e^{2\Lambda(\rho)}d\rho^2 + \rho^2 d\phi^2 + dz^2$ coincides with the Riemannian geometries [R_star] in Einstein's field equations: $R_{\mu\nu} - (1/2)R g_{\mu\nu} = 8\pi T_{\mu\nu}$. Here, the energy-momentum tensor $T_{\mu\nu}$ contains a perfect fluid $T_{\mu\nu}^{fluid} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$ and an electromagnetic component $T_{\mu\nu}^{EM} = F_{\mu\alpha}F_\nu{}^\alpha - (1/4) g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$, where ρ is the energy density, p is the pressure, u_μ is the four-velocity, and $F_{\mu\nu}$ represents the magnetic field $B(\rho)$. $B(\rho)$ is constrained by Maxwell's equations $\nabla_\mu F^{\mu\nu} = 0$.

In the weak field limit ($g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| << 1$), and in non-relativistic motion ($v/c << 1$) the geodesic equation, $d^2x^\mu/d\tau^2 + \Gamma^\mu_{\nu\lambda} (dx^\nu/d\tau)(dx^\lambda/d\tau) = 0$ becomes $d^2x^i/dt^2 = -\Gamma^i_{00} \approx -(1/2)\partial_i h_{00}$. The metric can be written as $h_{00} \approx -2\Phi/c^2$, where the gravitational potential Φ gives back the force per unit mass $F^i/m = -\partial_i \Phi$. The Laplacian (Poisson) equation $\nabla^2 \Phi = 4\pi G (\rho + 3p - B^2/8\pi)$ includes matter and magnetic contributions.

Lorentz-like force due to a magnetic field If $B = Hiz$, then for a particle with charge q and velocity v, $F = F_{mag} = qv \times B / c$ =. (4) This is the total Newtonian force: ($F/m = -\nabla (\Phi) + (q/m)(v \times B)/c$), so metric solutions will usually include Bessel functions from cylindrical symmetry. It relates to how one connects relativistic and Newtonian physics and elucidates particle dynamics in the magnetized cosmological spacetimes.

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